Comments on Theory for Extending Single-Product Production Function to Multi-product Settings

by

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Substantive Goal: Integrate multi-product firms into the analysis

- of productivity, and the
- analysis of markups that uses production data.

They start with an important fact

"Statistical agencies around the world are running production surveys though which they collect precise information about the products made by firms..."
"These datasets cover a large subset of mostly manufacturing firms and typically contains both values and quantities for each good produced by firms."

Some background on why this is important.

- There have been attempts to integrate multi-product production without sufficiently disaggregated quantity data, but the assumptions required are drastic.
- Most large firms are multi-product. In Belgium 75% of the revenue from manufacturing (using their eight digit classification of products) and no product is "dominant".
- As noted in O-P, if we do not analyze separate outputs, the estimates elasticities of output to input we get are elasticities of revenue ($p \times q$) to input.
- We do care about the causes & effects of changes in revenue given costs, but the variance in revenues is caused by changes in p as well as q changes. So the interpretation of these elasticities as production function elasticities is problematic.
- Manufacturing is 11-12% of GDP. In other sectors the multi-product nature of production is also central to tracing out the impact of /environmental changes (e.g. the change in patient allocations among hospitals after mergers).

Production vs revenue functions.

• Most analysis of production data to date focuses on the impact of environmental changes on revenue productivity. The analysis is careful to correct for endogenous input decisions and sometimes selection mechanisms (exit in establishment data, mergers among traded firms, privatized vs. public firms...). This reflects the fact that as economists our comparative advantage is the analysis of incentives: not in production processes per se.

- Revenue changes are generated by Δp as well as Δq, and the analysis did not distinguish between them.
- Not distinguishing between Δp & Δq made welfare analysis problematic & implied we could not analyze certain processes.
- E.g. in O-P we found (unweighted) average equipment industry productivity did not increase following the break-up of A.T. &T. All the measured productivity increase was a result of reallocation. The data misses the impact of the entry of competitors that followed the break-up on price declines.

So figuring out how to use the new data to analyze multi-product production would be helpful.

We were taught how to solve a related problem as undergraduates.

- Inputs determine the production possibility frontier (PPF) for output vectors, $T(q_1, \ldots q_m, x_1, \ldots x_n)$.
- A price (or marginal revenue) vector determines $q_1, \ldots q_n$.
- Inputs for each $q_1, \ldots q_n$ are chosen so that marginal revenue product equals marginal cost.

If we ignore uncertainty and assume inputs are purchased in competitive markets, the analogous firm's problem is

$$\max_{p_1, \ldots p_m, x_1, \ldots x_n} \sum_j p_j q_j(p_1, \ldots, p_m) - \sum_i c_i x_i, \text{ subject to}$$

$$T(q_1, \ldots q_m, x_1, \ldots x_n) \leq 0, \; q \in \mathcal{R}^m_+, \; \& \; x \in \mathcal{R}^n_+.$$

There is a set of $M + N + 1$ first order and complementary slackness conditions that determine the solution. The solution depends on *both* $T(\cdot)$ and the demand functions.

- For a fixed x there will generally be different combinations of $q_1, \ldots, q_n$ that can be produced (our PPF).
- So there does not exist a unique vector of functions mapping x into q. More precisely Γ^{-1} depends on demand parameters.
- I.e. there may well be production functions

$$q_j = \sum_k x_{i,k}\beta_{j,k} + \sum_{r \neq i} q_r \gamma_{j,r} + u_j.$$

However if if $x_k \equiv \sum_j x_{j,k}$, there do not exist functions

$$q_j = \sum_k x_k \beta_{j,k} + \sum_{r \neq i} q_r \gamma_{j,r} + u_j$$

with invariant coefficients because the division of x_k among the q_j depends on the demand functions.

• *The problem:* though we observe $q_1, \ldots, q_M$, we do not observe the $\{x_{k,j}\}$, the inputs allocated to each output.
• Assuming no demand functions from elsewhere how do we proceed? There are two sources of information and two sets of parameters (from demand, and from $T(x)$). The info:

1. the $M + N + 1$ first order optimality conditions provide information on demand and production parameters.
2. the values of parameters that are consistent with the observed $(x_1, \ldots x_n)_t$ and $(q_1, \ldots q_m)_t$ provide info on production parameters.

Estimation needs to integrate disturbances and worry about endogenous input choices & possibly selection. Possibilities:
• Control functions: need unique u vector for each control vector,
• Instruments: search for the θ which make the residual distributions independent of IV (prices for productivity residual?).

Some implications for what has been done.

- Even if goods are not complements or substitutes in production (so $\gamma_{j,k} \equiv 0$), the map from aggregate inputs to outputs still depends on the demand parameters.
- If, in addition, the goods are not substitutes or complements in demand and we let $\epsilon_{i,j} \equiv \partial log(q_j)/\partial log(x_{i,j})$ & $mc_j =$ marginal cost$_j$, the equation for markups is

$$\frac{c_i x_i}{\sum_j p_j q_j} = \sum_j \frac{mc_j}{p_j} \frac{p_j q_j}{\sum_j p_j, q_j} \epsilon_{i,j}.$$

  So even in this simple case we can not obtain markups without output elasticities for the individual products.
- This case has no interactions among products in production or demand, so it rules out most reasons for multi-product firms, and we know the economy is dominated by them.
- We need to develop the empirical analysis further to be able to make statements about markups from production data for the vast majority of the economy.

Conclude.

The authors deserve accolades for

- opening up an important research program,
- showing us that data is available to do it in manufacturing, and my guess is in other sectors as well, and
- demonstrating that this type of research is needed before we can either evaluate the production effects of environmental changes or interpret productions data to explain historical phenomena.

• With such an ambitious project it should not be surprising that there are issues still to be resolved.